SIGGRAPH ASIA 2024 TOKYO 東京

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Dynamic skeletonization via variational medial axis sampling

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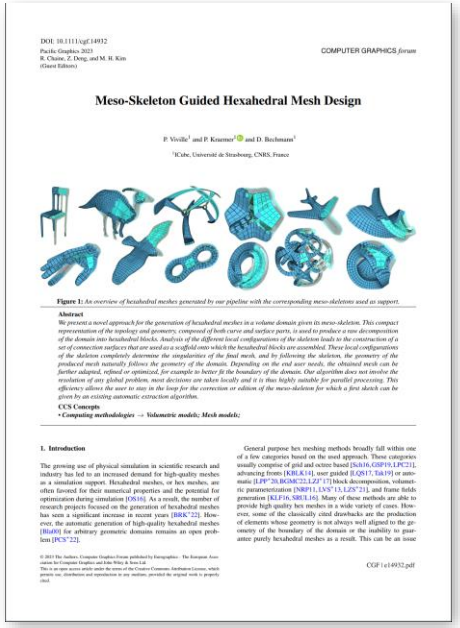
Skeleton: Valuable Tool

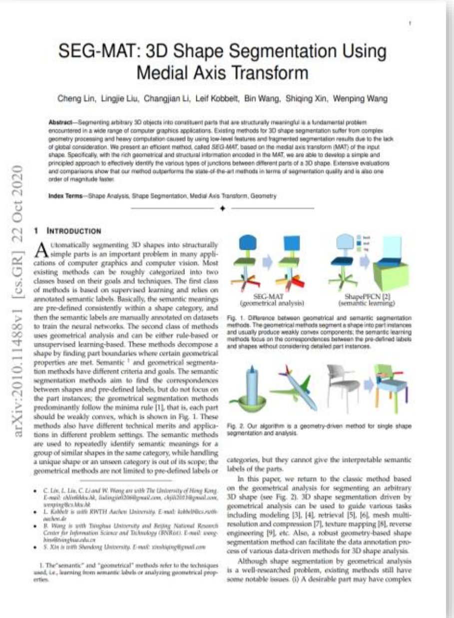
Simulation

[Lan et al. 2020]



Mesh Generation

[Viville et al. 2023]

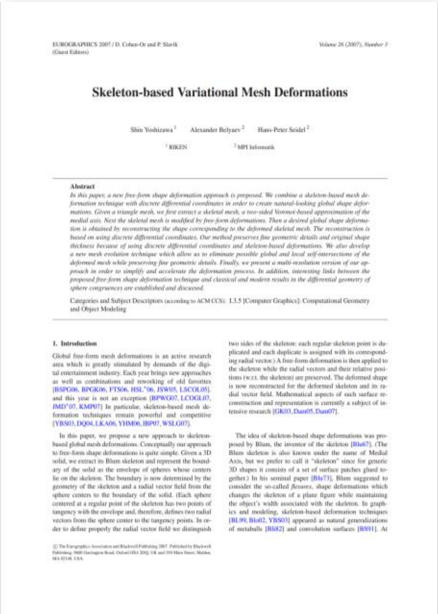


Segmentation

[Lin et al. 2020]

Single Image Reconstruction

[Hu et al. 2022]

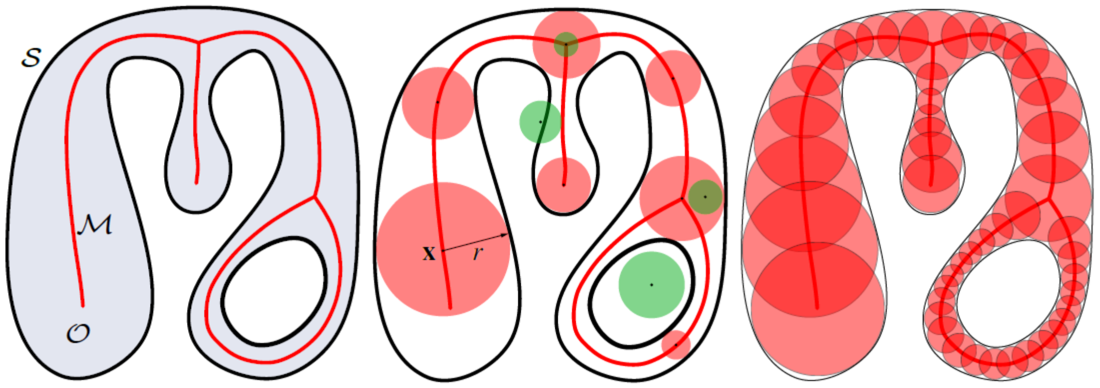


Deformation

[Shin et al. 2007]

Skeleton: Discretized Medial Axis

- **Medial Axis:** The set of centers of spheres that have at least two closest points on the boundary of the shape. Such a sphere is called a ***medial sphere***

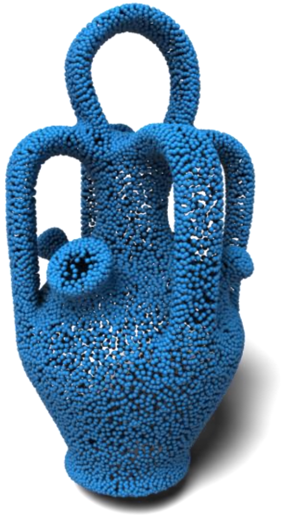


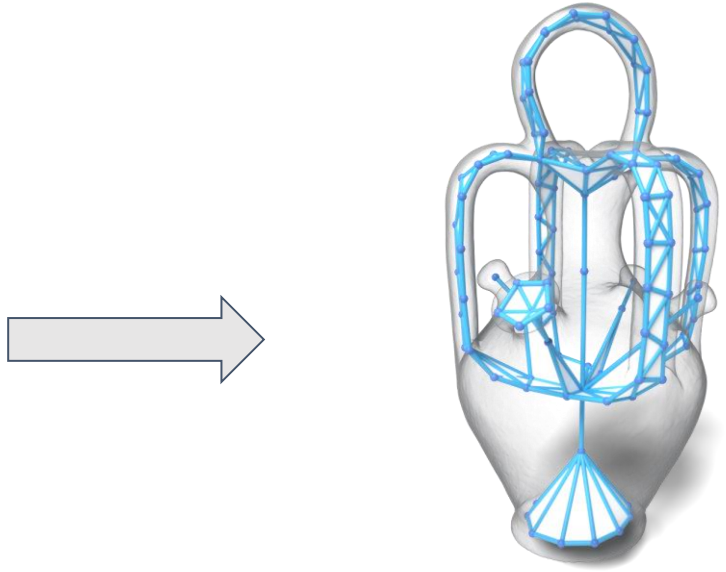
[Tagliasacchi et al. 2016]

Objective

Surface

Oriented Point cloud





Discretization of Medial Axis

Medial Axis Approximation

Voronoi Diagram
[Amenta et al. 1998]

Shrinking balls
[Ma et al. 2013]

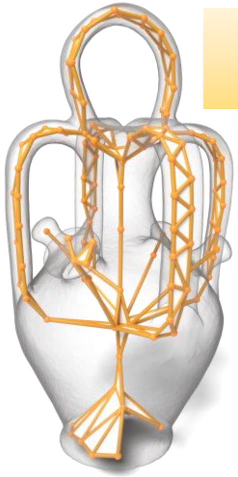
Random sampling
[Dou et al. 2022][Wang et al. 2024]

Impratical for application

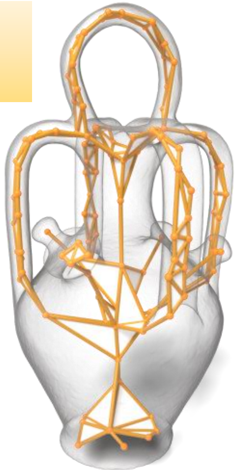
Simplification
(Fine to Coarse)

Refinement ?

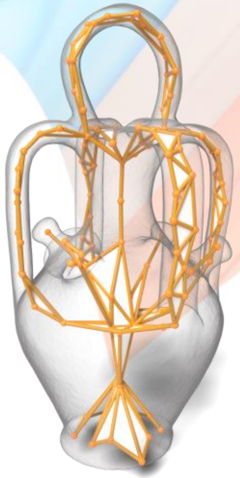
Q-MAT
[Li et al.2015]



Coverage Axis
[Dou et al. 2022]



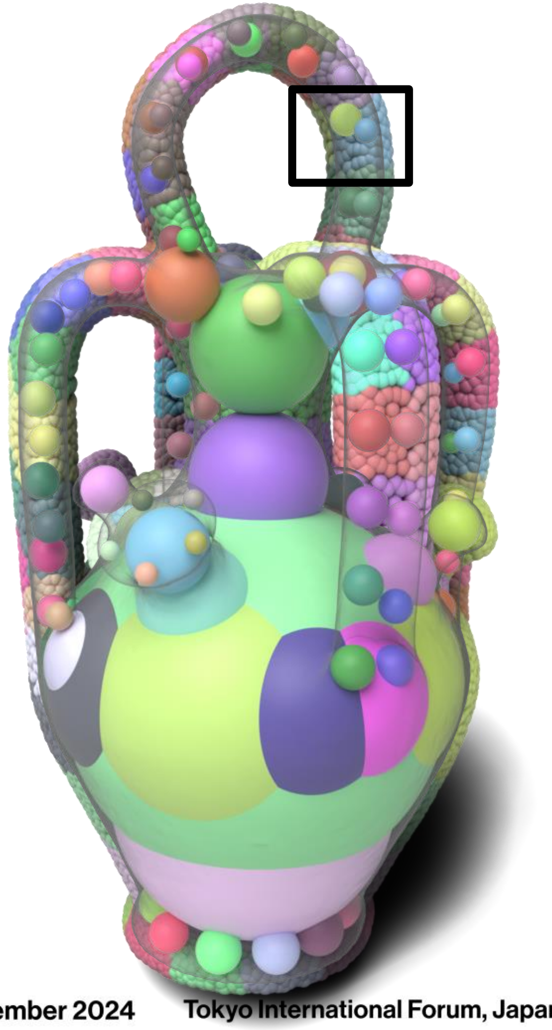
Coverage Axis++
[Wang et al. 2024]

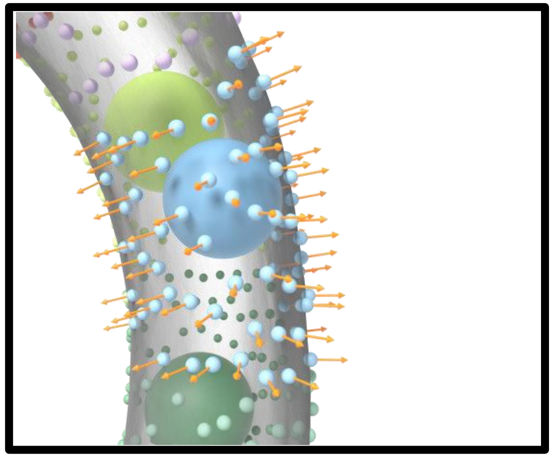


- Feature preservation
- Time consuming
- No control on the result
- Irregularity of the distribution of medial samples

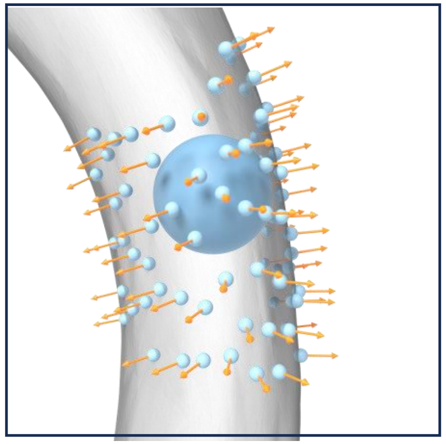
Method

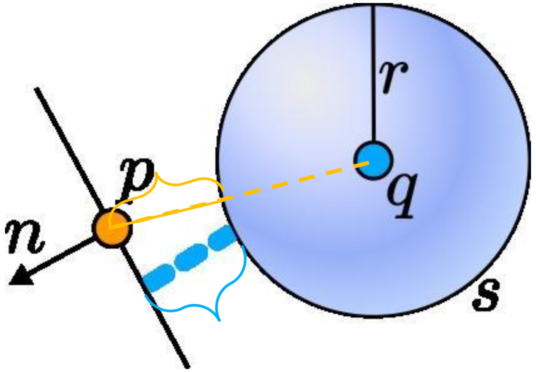
Observation: Each medial sphere occupies a segment of surface





Metric





- Sphere-plane distance:

$$d_{p,n}(s) = n^t \cdot (p - q) - r$$

- Spherical quadric error metric: *[Thiery et al. 2013]*

$$d_{p,n}(s)^2 = Q_{p,n}(s) = \frac{1}{2} s^t \cdot A \cdot s - b^t \cdot s + c$$

- Diffused quadric:

$$Q_{v_i}(s) = \sum_{t_j \in T(v_i)} \frac{\mathcal{A}(t_j)}{3} Q_{v_i, n_j}$$

- Sphere-point distance:

$$D_{v_i}(s) = \left(\sum_{t_j \in T(v_i)} \frac{\mathcal{A}(t_j)}{3} \right) (|p - q| - r)^2$$

$\mathcal{A}(t_j)$: area of triangle (KNN graph for point cloud)

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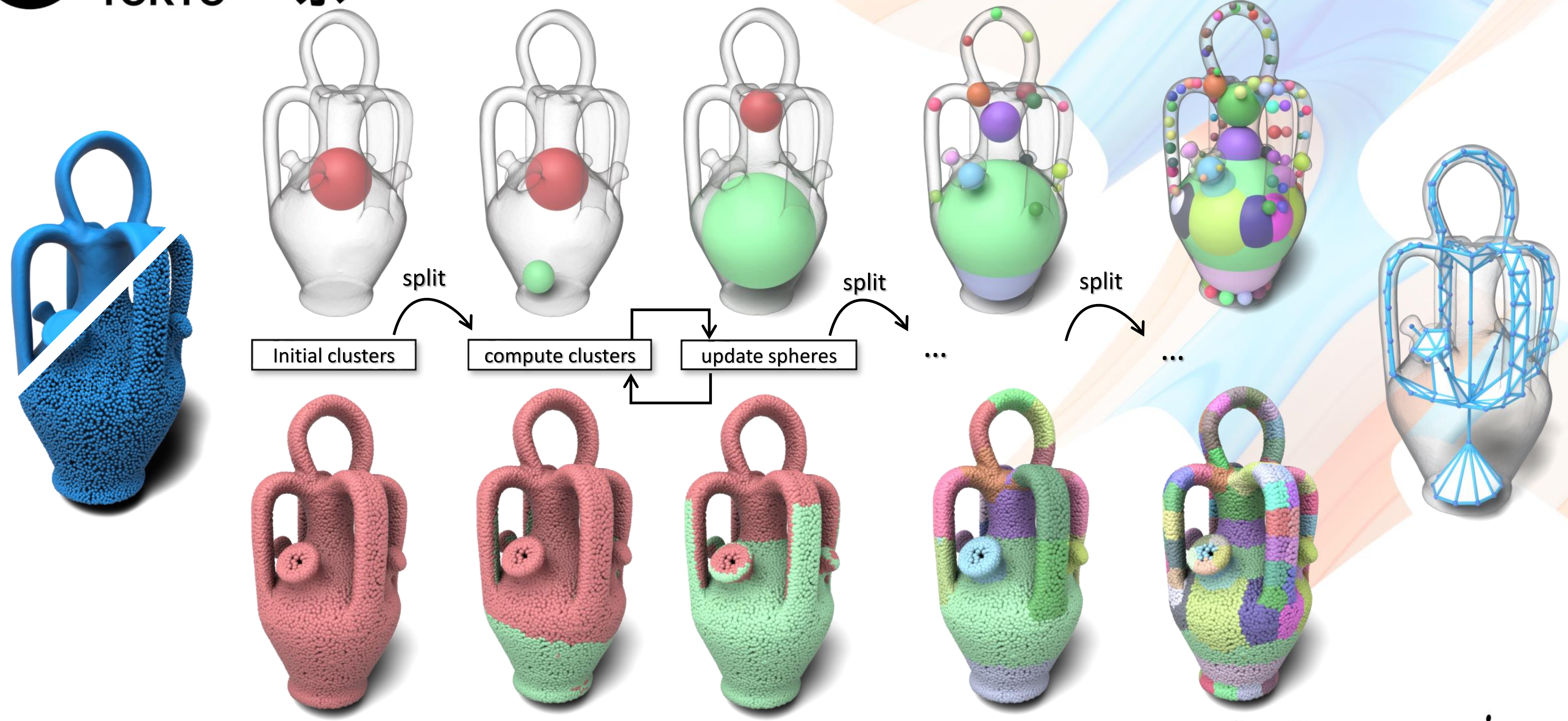
Pipeline



Compute clusters

$$E_{v_i}(m_j) = Q_{v_i}(m_j) + \lambda D_{v_i}(m_j)$$

$\lambda = 0$ $\lambda = 0.02$ $\lambda = 0.2$ $\lambda = 1$

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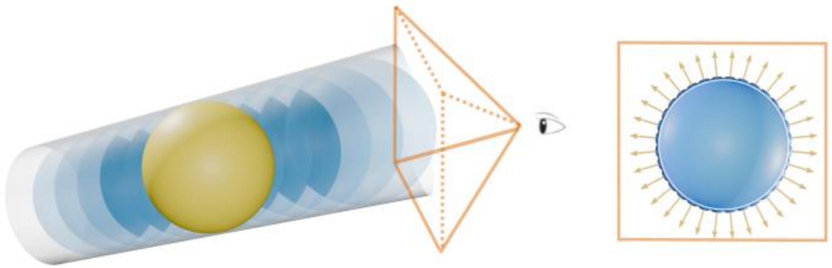
Update spheres

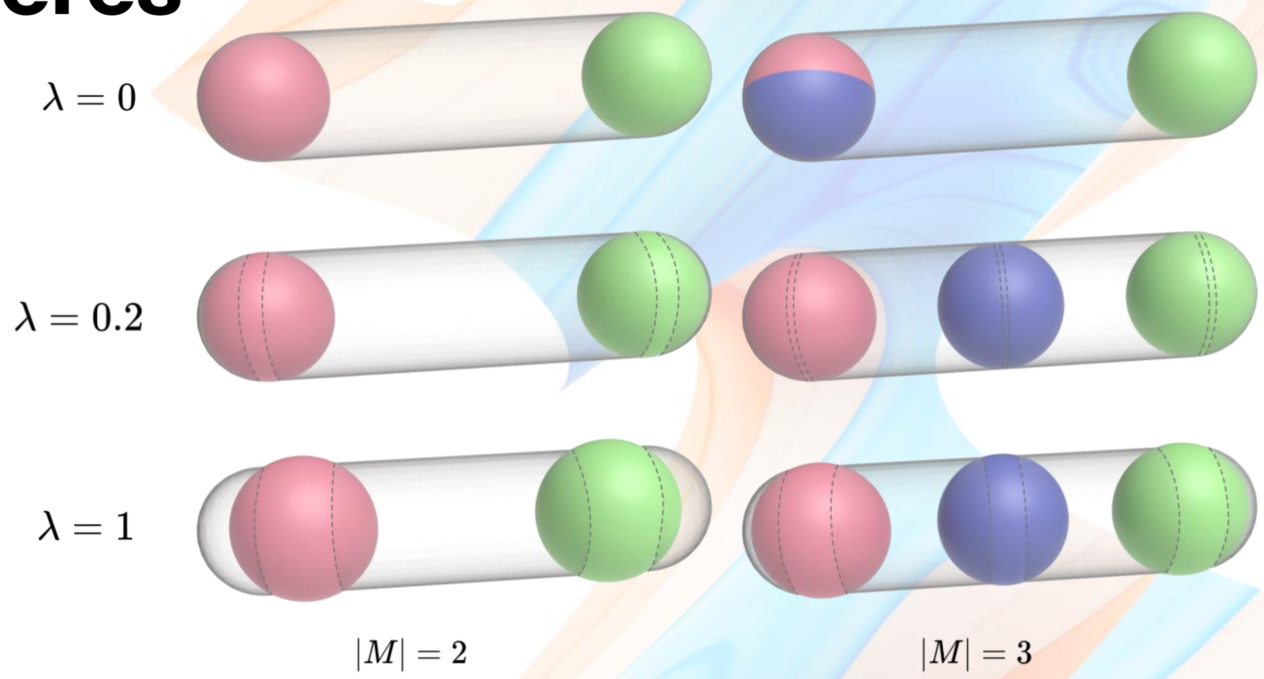
- For each cluster vertices, fitting a sphere which minimizes the following metric:

$$(q_i^*, r_i^*) = \arg\min_{q_i, r_i}\Big(E_{SQEM}(\mathcal{C}_i) + \lambda\, E_{euclidean}(\mathcal{C}_i)\Big)$$

$$E_{SQEM}(\mathcal{C}_i) = \sum_{v_j \in \mathcal{C}_i} Q_{v_j}(m_i)$$

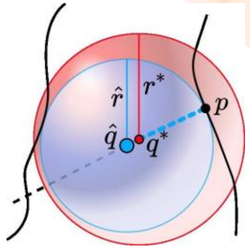
$$E_{euclidean}(\mathcal{C}_i) = \sum_{v_j \in \mathcal{C}_i} D_{v_j}(m_i)$$





- No guarantee that the optimized sphere is medial sphere or within the shape

- Sphere Projection: Project the sphere center on the medial axis in the direction of the gradient of distance function.



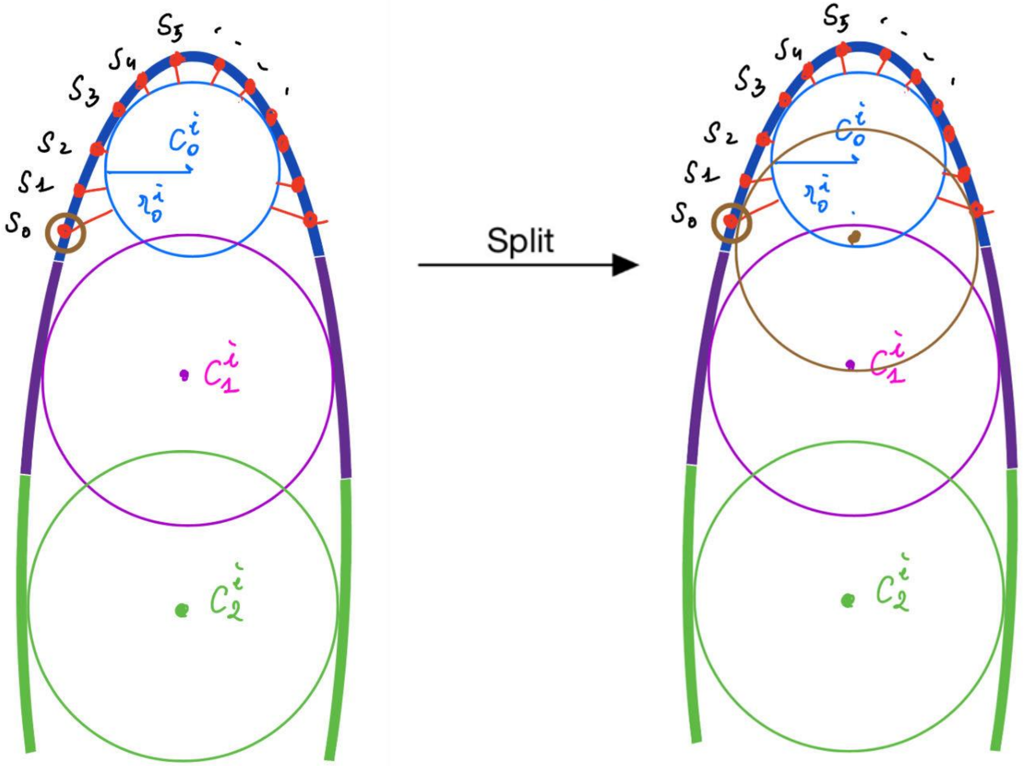
Sphere splitting

- For each cluster C_i evaluate the error to determine whether it should be split.

$$E(C_i) = \frac{1}{\mathcal{A}(C_i)} \sum_{v_j \in C_i} E_{v_j}(m_i)$$

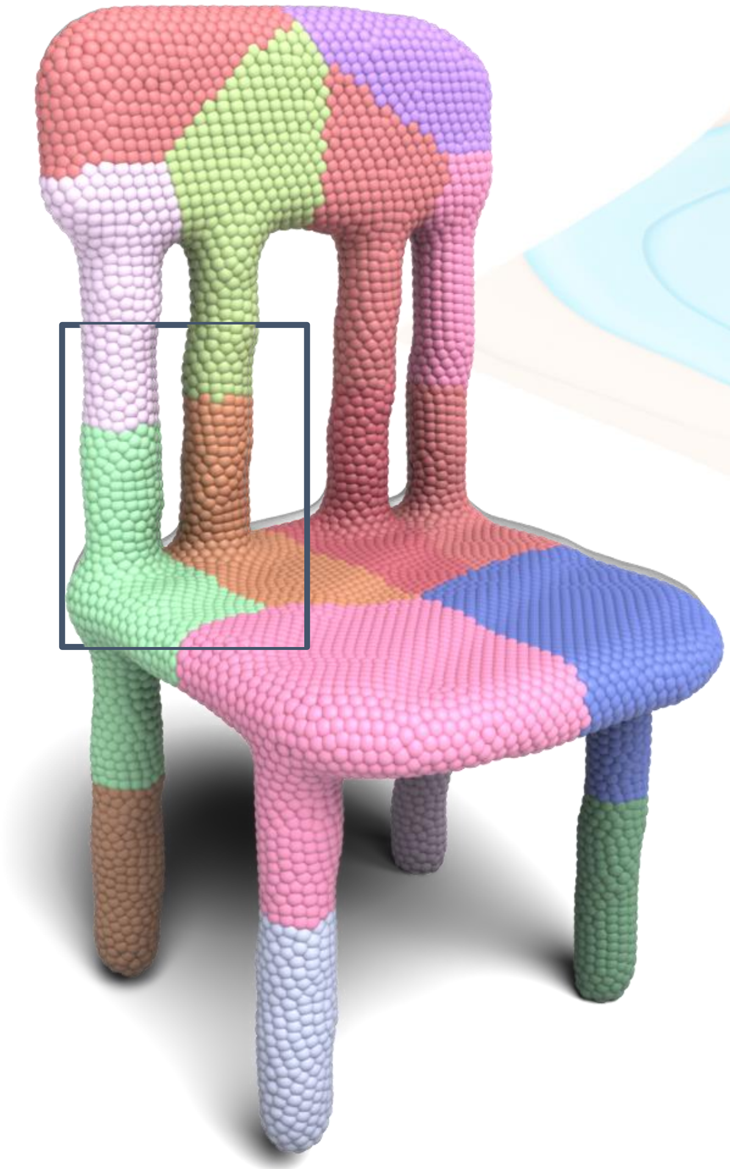
- Taking the vertex that has largest error as a seed to create a new sphere

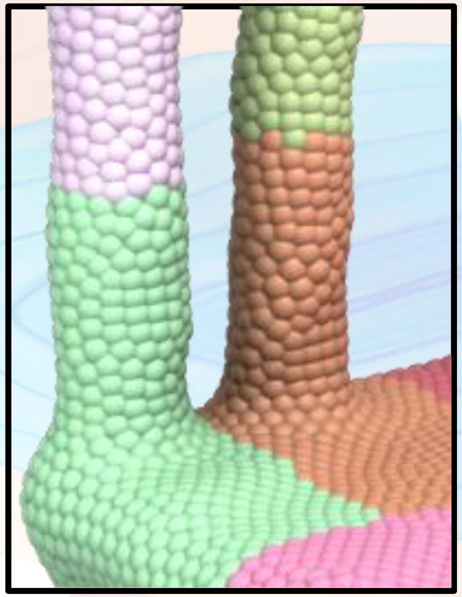
$$v_{max} = \arg\max_{v_i \in C_i} E_{v_j}(m_i)$$

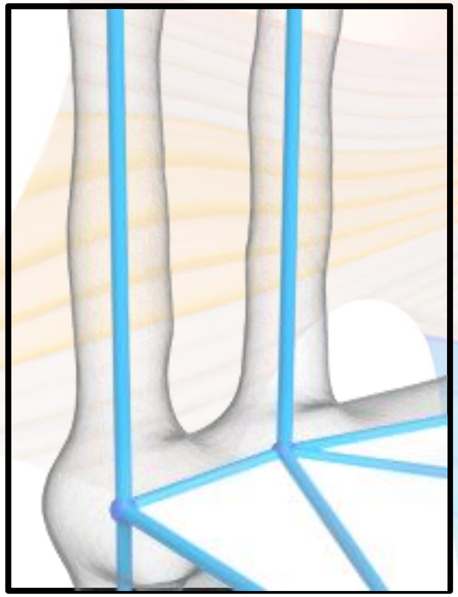


Connectivity

- Build edge if two clusters are adjacent.
- Build face if three clusters share the same neighbours







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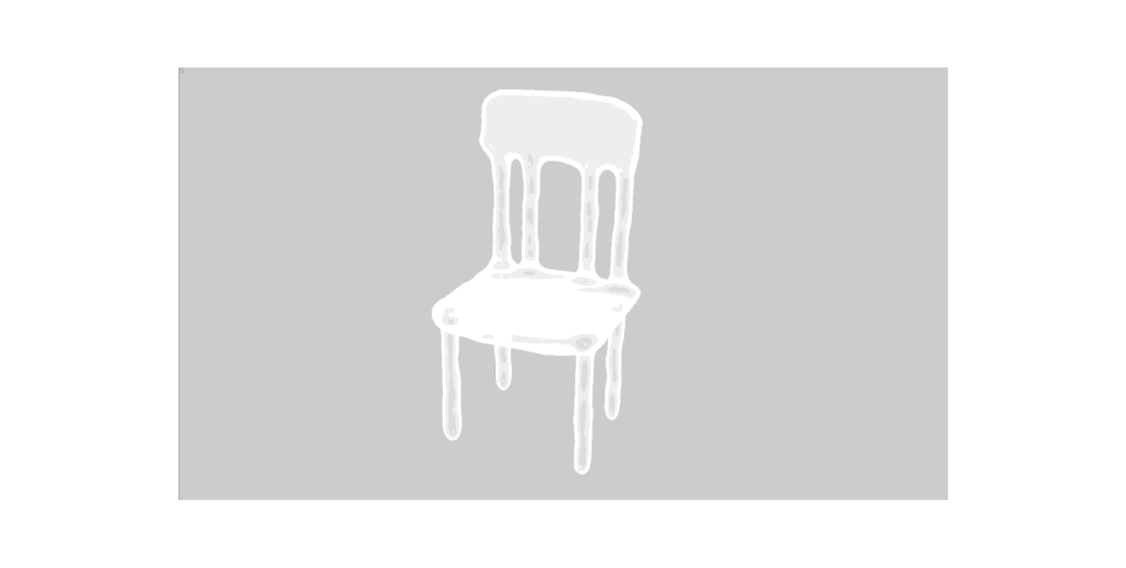
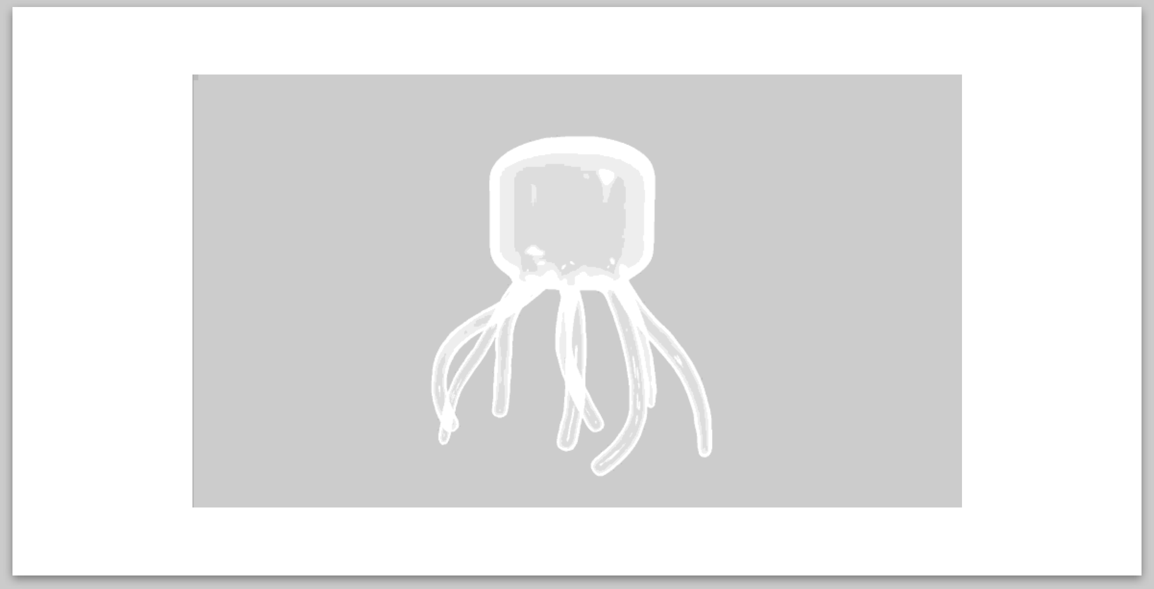
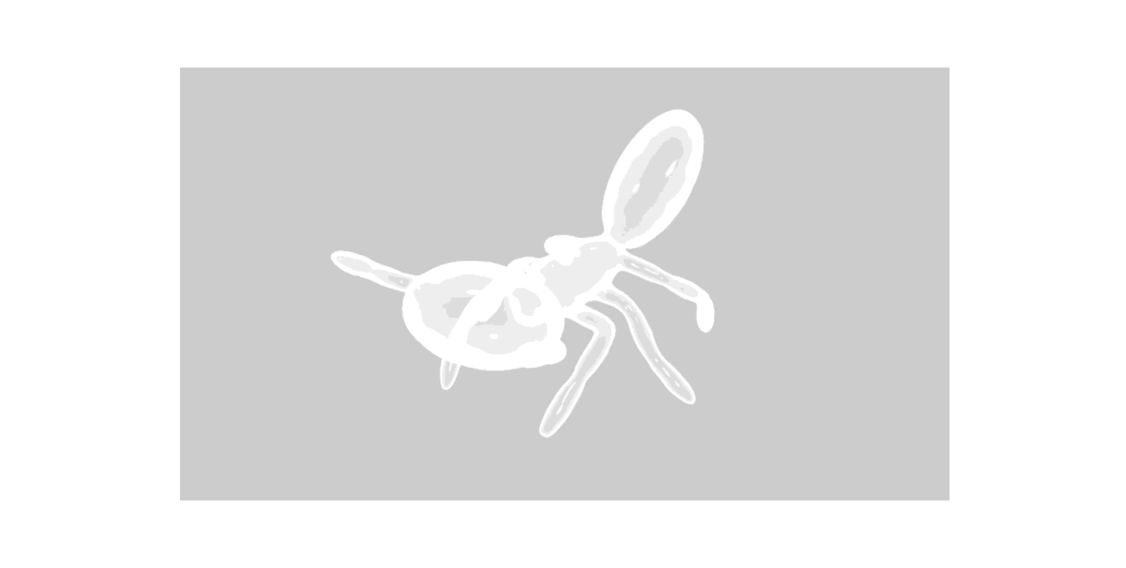
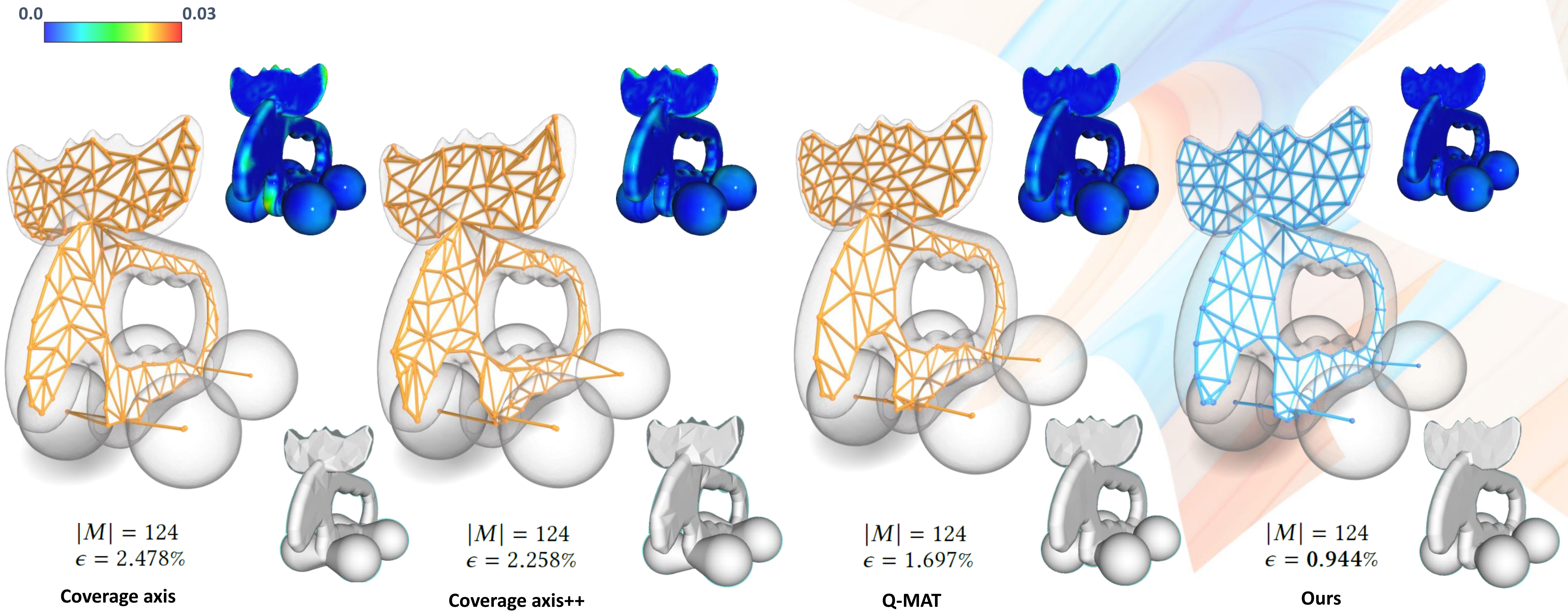
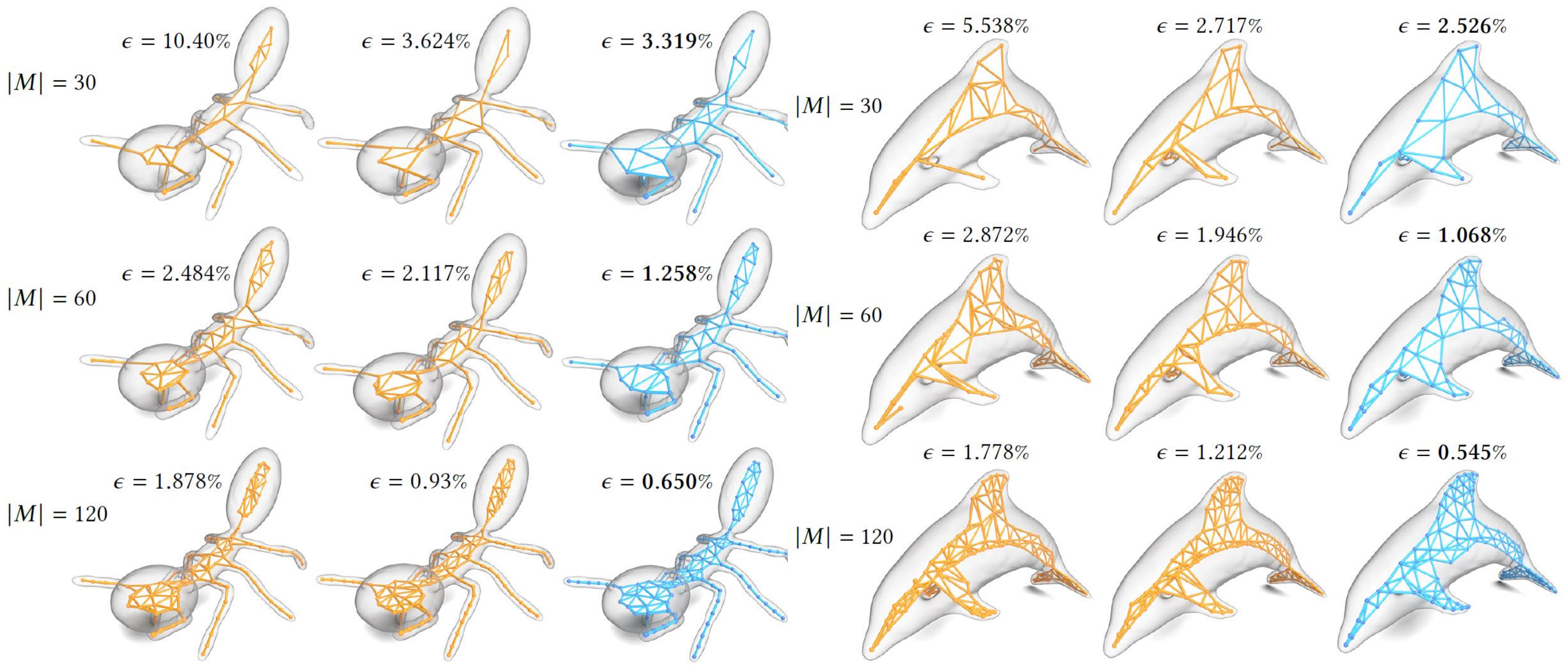
Result


Visual Comparison

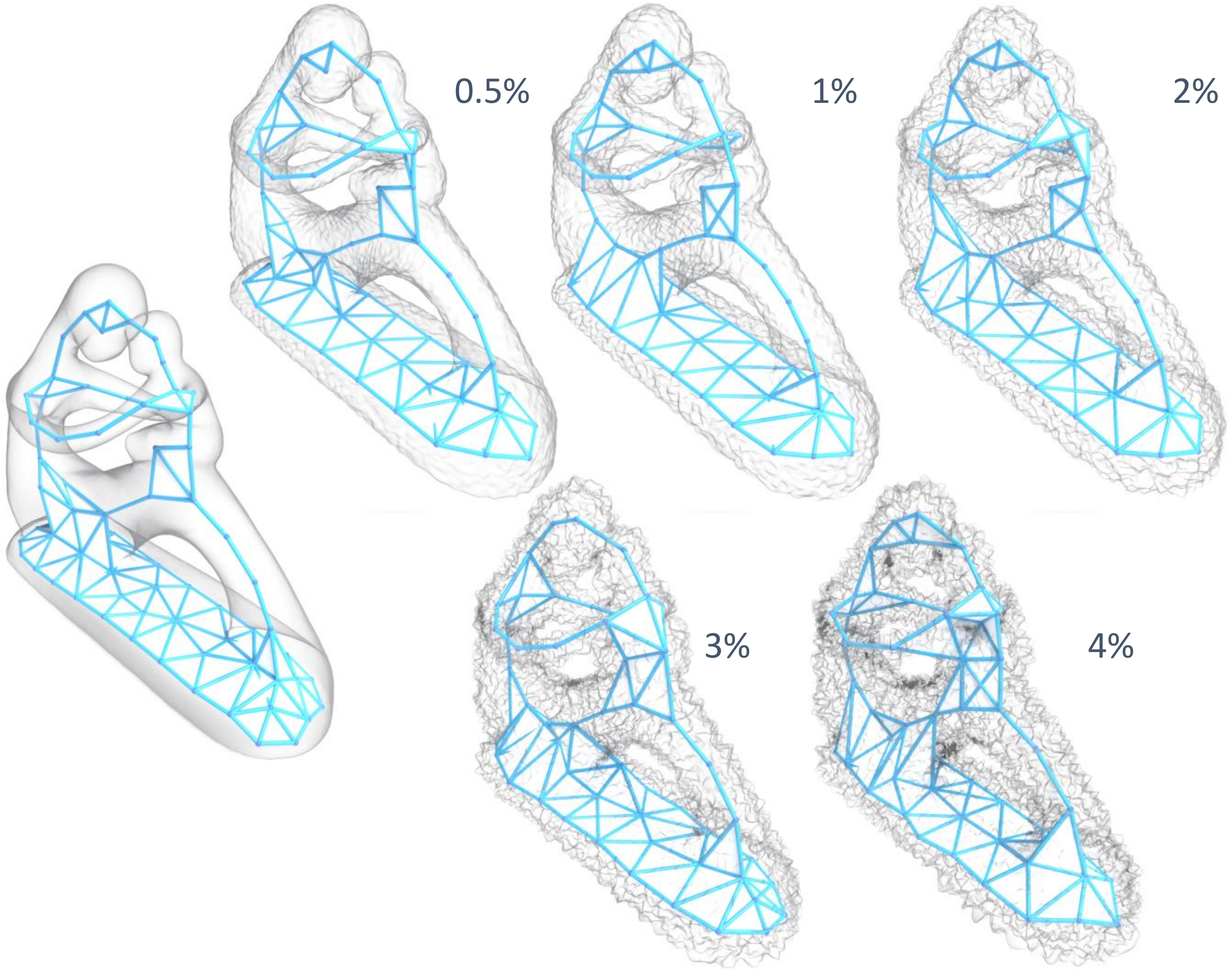


Comparison: Different resolution

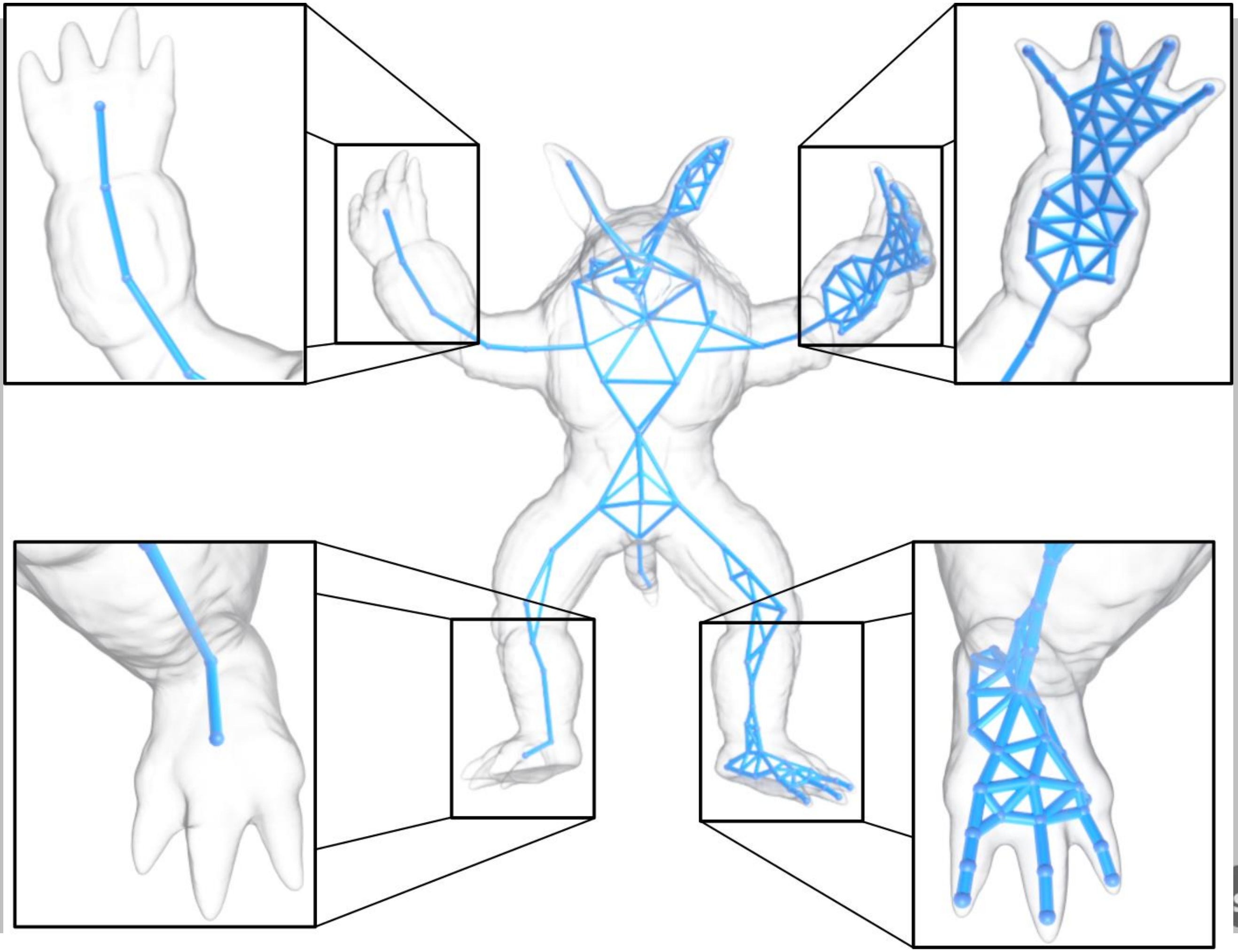


Coverage Axis ++ **Q-Mat** **Ours** **Coverage Axis ++** **Q-Mat** **Ours**

Robustness to noise



Interactive edition of skeleton

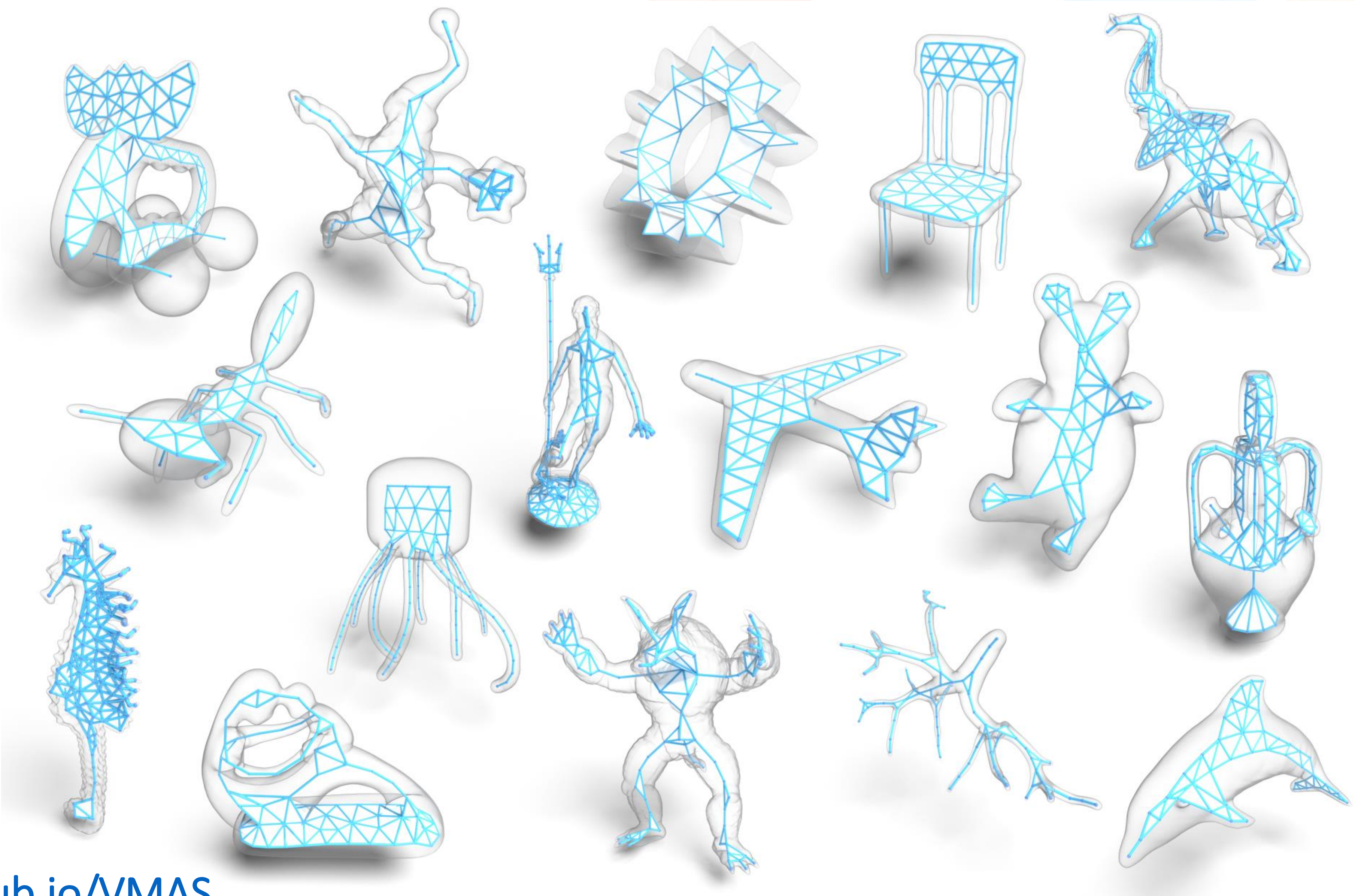


Limitation and future work

- **Limitation:**
  - **No Global Convergence**: There is potential for oscillations in the positions of medial spheres.
  - **Topology Mismatch**: Coarse resolutions may result in a topology that differs from the input shape.
  - **Suboptimal connectivity**: Intersecting triangles or closed surfaces may occur.

- **Future work:**
  - **Medial Sample Freezing**: Lock samples in place to improve control.
  - **Adaptive Density Function**: Enable region-specific refinement.
  - **Support Diverse Inputs**: Extend to binary images or incomplete data.

Gallery



Project page:
https://huang46u.github.io/VMAS
Code will release soon!